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Tutorial on infinite panel sound transmission loss simulations

Stephen A. Hambric[a)]
Hambric Acoustics, LLC

Infinite panel theory often simulates finite panel sound transmission loss (TL) quite accurately, even for panels with complex cross sections. This tutorial explains how to easily compute infinite panel TL for different angles of sound incidence. The tutorial also explains how to compute statistical averages of transmitted power due to impinging sound with random angles of incidence. Concepts like coincidence frequency, and its corresponding TL degradation; low-frequency mass law behavior; and the effects of adding damping to a structure are explained. Images of incident and radiated sound, along with panel vibrations, are used to illustrate the basic theory.

1 INTRODUCTION

Sound power transmission loss (TL) is simulated and measured for many types of noise barriers, including windows, doors, walls, and enclosures designed specifically to mitigate sound from noisy machinery. Expensive computational models are often constructed and analyzed to estimate TL. TL measurements are also expensive, and can be corrupted by background noise from adjacent noise sources. Before complex models or measurements are pursued, basic infinite panel TL theory should always be exercised to estimate TL, and approximate the effects of design modifications which add damping or mass. This tutorial explains basic infinite panel TL theory, and is an expanded version of the material in the general sound-structure interaction tutorial by Hambric and Fahnline[1] which appeared in *Acoustics Today* Magazine in 2007.

a) email: hambricacoustics@gmail.com

2 INFINITE PANEL TRANSMISSION LOSS THEORY

2.1 Single angle of incidence

Consider the classical problem of a flat infinite panel that is struck by an incoming acoustic wave. There are three pressure waves next to the panel surface – the incident wave, a reflected wave, and a wave re-radiated by the structure, which has been forced into vibration by the incident and reflected waves. The sum of the incident and reflected waves forms a 'blocked' pressure on the surface, and if the surface is rigid, the blocked pressure field is, in fact, the total pressure. However, if the structure is flexible and vibrates, it radiates a third pressure wave, which sums with the blocked pair to form the overall pressure field.

Figure 1 shows the incident and reflected waves for a 30 degree angle of incidence (the angle is taken from the direction normal to the plate), along with the total blocked pressure field acting on a rigid surface. Notice how the two waves combine to form a standing wave pattern in the direction normal to the surface. In this example, the standing wave pattern of total blocked pressure propagates upward in the direction parallel to the surface at a speed $c_o\sin(\phi)$, where c_o is the acoustic wave speed and ϕ is the angle of incidence.

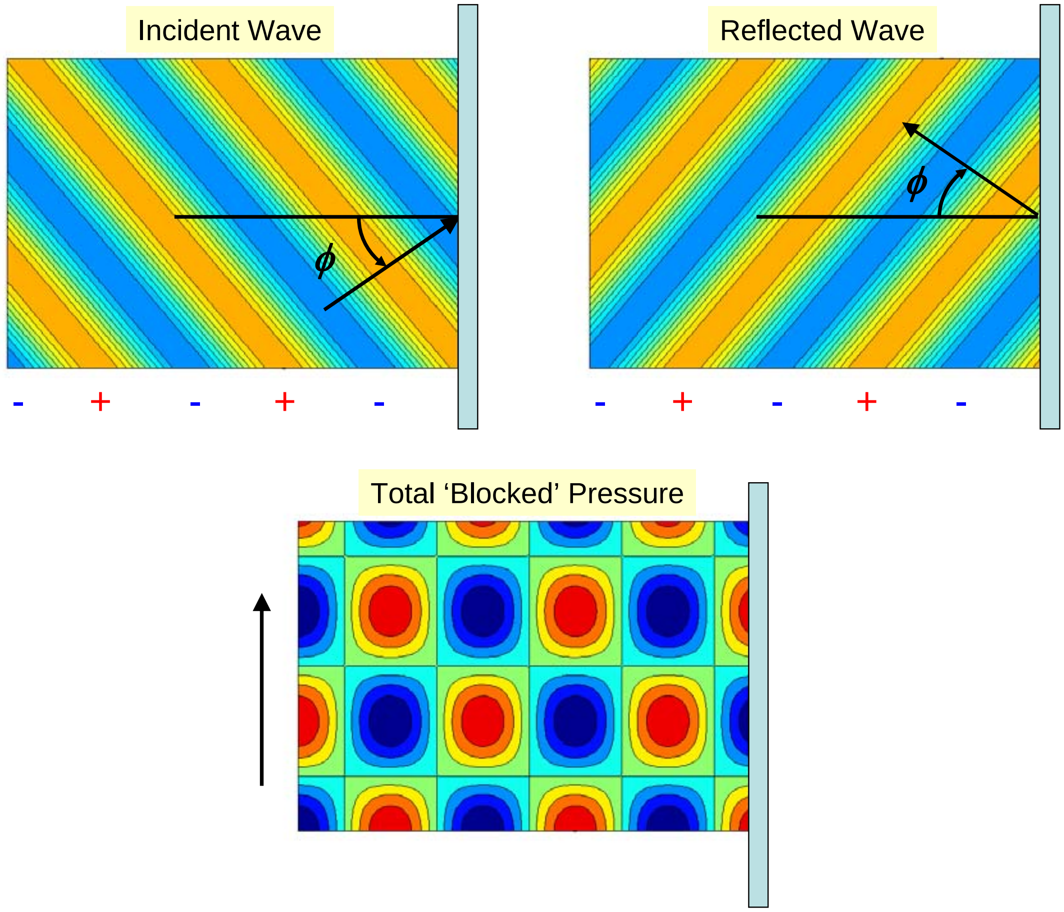

Fig. 1 – Blocked pressure field acting on a plate (right side of images) at a 30 degree angle of incidence.'+' and '-' signs indicate phase variations in the waves.

Infinite plate theory can be used to estimate the plate vibration caused by the blocked pressure field:

$$v_n = \frac{2\,p_{incident}}{z_{fluid} + z_{plate}} \tag{1}$$

where z_{fluid} and z_{plate} are the impedance (pressure/velocity) of the acoustic space and the plate. The fluid impedance depends on the pressure wave's angle of incidence and the fluid's characteristic impedance:

$$z_{fluid} = \rho_o c_o / \sqrt{1-\sin^2\phi_{incident}}\ , \tag{2}$$

and the infinite plate's impedance at frequency ω is

$$z_{plate} = D(k_o \sin\phi_{incident})^4 \eta / \omega - i\left[D(k_o \sin\phi_{incident})^4 - \rho h \omega^2\right] / \omega\ , \tag{3}$$

where D is the flexural rigidity, η is the structural loss factor, ρ is the panel mass density, h is the plate thickness, and k_o is the acoustic wavenumber ω/c_o.

Examining the equation shows that the plate vibrates most when its structural impedance is minimized. This occurs when the stiffness and mass terms in the plate's impedance cancel each other, or when $D(k_o \sin\phi_{incident})^4 = \rho h \omega^2$. To find this frequency, we replace k_o with ω/c_o, and find that:

$$\omega_{co} = \sqrt{\frac{\rho h}{D}}\frac{c_o^2}{\sin^2\phi_{incident}} \tag{4}$$

ω_{co} looks similar to a plate's critical frequency, and to its critical radiation angle for frequencies above coincidence. In fact, when $\phi_{incident}$ is 90 degrees (where the acoustic waves propagate in the plane of the plate, or 'graze' the plate), the coincidence frequency is actually the critical frequency. The critical frequency is sometimes called the lowest coincidence frequency. This means that there is no single frequency where the plate vibrates most – there are many of them, each of which corresponds to a different angle of incidence.

Now, what happens when there is also fluid on the other side of the plate? How much of the incident sound gets through the plate to the other side? This is the classic *sound transmission loss* (TL) problem, and may be solved easily for an infinite plate, and not so easily for a finite one. Fahy and Gardonio[2] provide a derivation of the sound power transmission coefficient through an infinite plate in their textbook, and we repeat the result here (assuming the fluids on both sides of the plate are the same):

$$\tau(\phi,\omega) = \frac{\left[2\,\rho_0\,c_0/\cos\phi\right]^2}{\left[2\rho_0 c_0/\cos\phi + (D/\omega)\eta\left(k_0\sin\phi\right)^4\right]^2 + \left[\omega\rho h - (D/\omega)\left(k_0\sin\phi\right)^4\right]^2}\ . \tag{5}$$

The η, ρh, and D terms in the equation represent the damping, surface mass density, and stiffness (flexural rigidity) of the plate, respectively. The amount of sound transmitted depends on the fluid properties, the structural properties, frequency (recall $k_o=\omega/c_o$), and the angle ϕ of the incident pressure wave with respect to the plate.

Some typical transmission loss (TL) plots for a thin metal panel, computed as the logarithm of the inverse of the transmission coefficient, $10\log_{10}(1/\tau)$, are shown in Figure 2. For acoustic waves not normally incident to the plate, sharp dips appear in the transmission loss. These dips correspond to sharp peaks in the transmission coefficient, and act as strong pass-bands of incident sound. The dips are at the coincidence frequencies of the plate. Recall from Equation 4 that the coincidence frequencies depend not only on the plate, but on the angle of incidence of the sound waves. As the angle of incidence changes, the coincidence frequency and the frequency of the transmission loss dip changes as well. Note that there is no coincidence dip for normally incident waves, since the acoustic waves do not propagate along the plate (more on this later).

At low frequencies, the mass term in equation 5 determines the transmission loss, which increases with the square of frequency (6 dB/octave, or 6 dB for each doubling of frequency). At high frequencies, above the coincidence dip, plate stiffness is dominant, and the transmission loss increases with the 6^{th} power of frequency, or 18 dB/octave. Figure 2 shows what most people already know from experience – it is hard to keep low-frequency sounds from propagating through barriers. Consider this the next time you close a door to block out sound from a hallway or another room. You stop hearing mid and high frequency sounds, but still hear 'muffled' low-frequency noise.

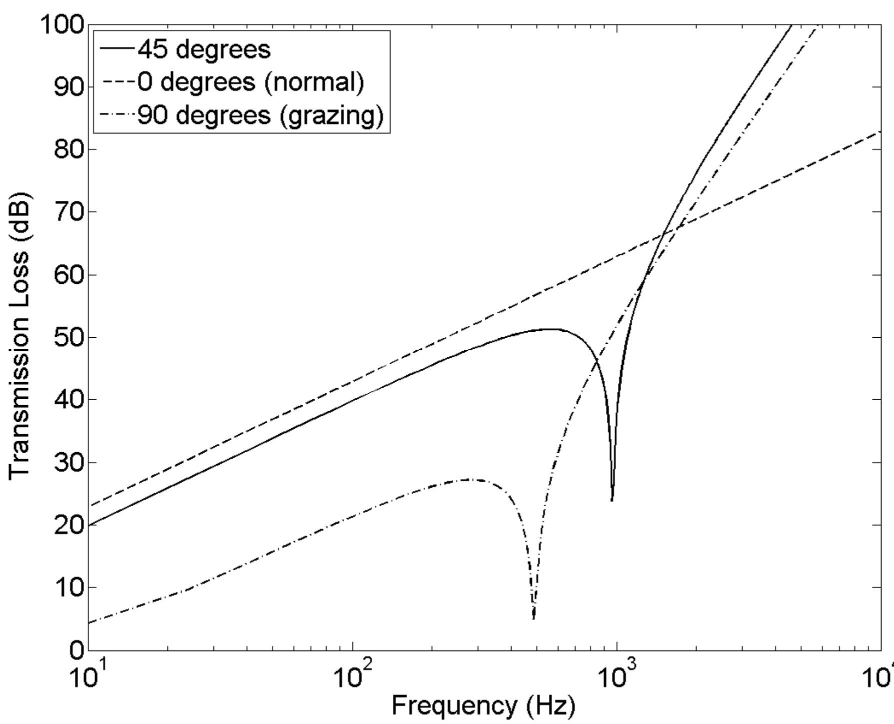

Fig. 2 – Typical transmission loss plot for variable angles of incidence. Grazing incidence refers to acoustic waves that are nearly in the plane of the plate.

To visualize the sound field incident on and transmitted by an infinite plate, Figure 3 compares pressure and displacement of a plate at two conditions: well below, and near coincidence. In the example, I have set the plate loss factor equal to 0. Try setting loss factor to zero and computing the transmission coefficient in Equation 5 at coincidence (remember, this is where the acoustic wavenumber in the plane of the plate matches the free bending wavenumber in the plate, or $k_o \sin\phi = k_b$). You should compute a transmission coefficient of 1, which is perfect sound transmission!

The strength, or depths of the coincidence dips depends strongly on the plate's loss factor η. Designers of noise barriers (windows, doors) try to minimize the depth and breadth of the coincidence dips. The most common approach for mitigating coincidence dips is using constrained layer damping, or CLD. Automotive glass in luxury vehicles, and glass in high-end office buildings usually have a thin layer of clear vinyl sandwiched between two panes of glass to increase structural damping. You'll learn more about the effects of damping in Section 2.2.

For zero, or normal angle of incidence (sound waves normal to the plate's surface), the transmission coefficient is not indeterminate (you might think it would be, since there are several terms in Equation 5 that divide by sin(ϕ)). The transmission coefficient for normal incidence actually simplifies to:

$$\tau\left(\phi = 0, \omega\right) = \frac{1}{1 + \left(\frac{\omega \rho h}{2\rho_o c_o}\right)^2} \tag{6}$$

which corresponds to the well-known 'mass law'. The mass law transmission loss is the highest curve (normal incidence) in Figure 2, and increases with the square of frequency over all frequencies, never showing a coincidence dip.

To improve transmission loss, many noise control engineers use two panels, particularly in windows. Called 'double glazing', the panels are separated by an air (or other gas) gap. Since transmission loss is additive (just add the TL values, or multiply the transmission coefficients together before computing TL), there is a substantial improvement, much more so than simply increasing the thickness of a single panel. However, most double panel systems have different panel thicknesses so that the coincidence dips of the two panels occur at different frequencies. While this leads to two pass-bands for incident sound, it is generally preferable to have two weak pass bands rather than one very strong one.

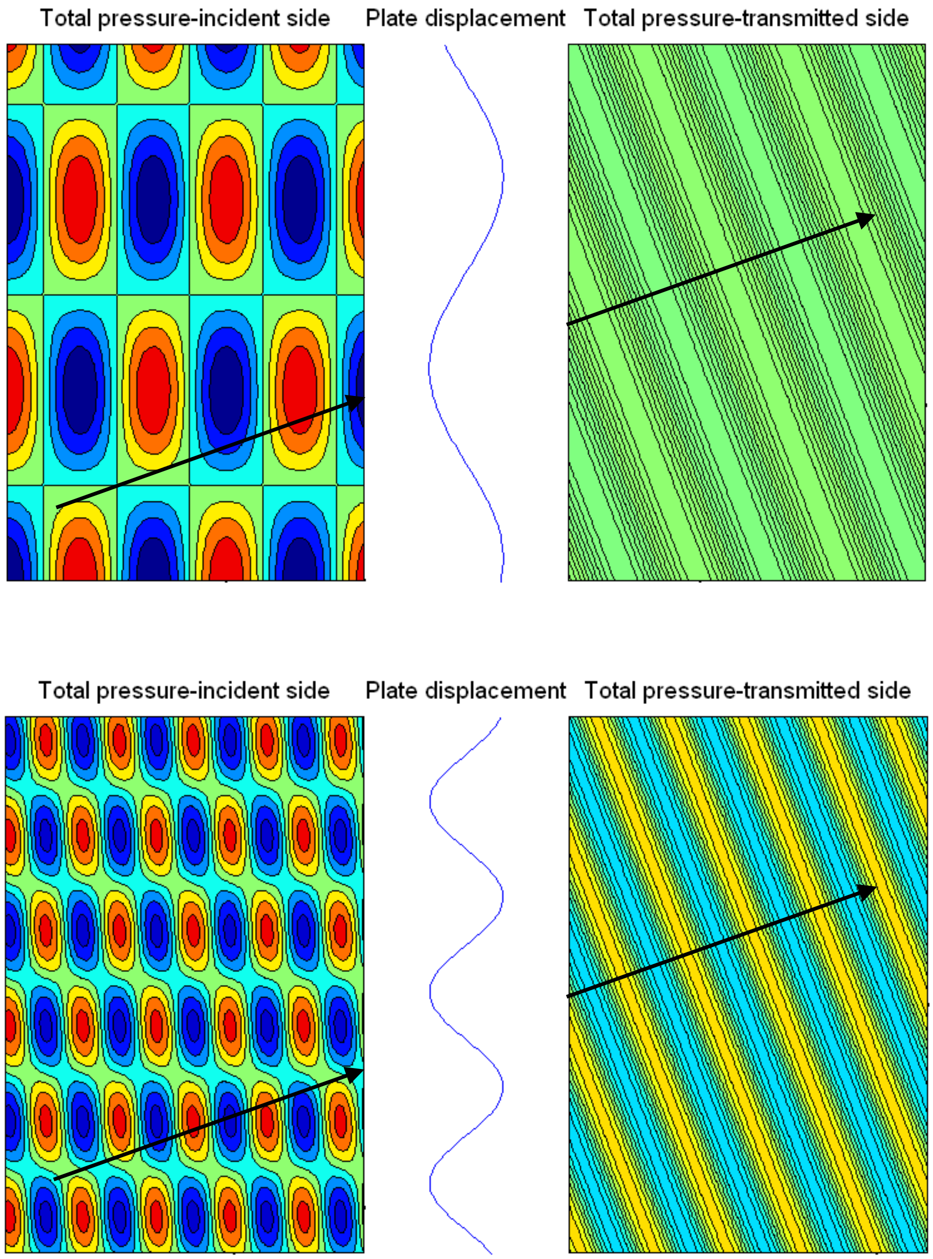

Fig. 3. Incident and transmitted sound fields around an infinite 25 mm thick steel plate, 30 degree angle of incidence. Top – at 50% of coincidence, Bottom – at coincidence.

2.2 Multiple random angles of incidence

In most practical situations, acoustic waves impinging on a panel do not always arrive from the same angle. Consider a window in an office building or a hotel. Sound waves arrive from all angles, with a random distribution of the angles over time (imagine incident sound from passing airplanes and automobiles, reflecting off of adjacent buildings). Therefore, statistical integration over all incidence angles is used to estimate a *random angle of incidence* transmission loss:

$$\tau_d = \frac{\int_0^{\frac{\pi}{2}} \tau(\phi)\sin\phi\cos\phi\, d\phi}{\int_0^{\frac{\pi}{2}} \sin\phi\cos\phi\, d\phi} = \int_0^{\frac{\pi}{2}} \tau(\phi)\sin 2\phi\, d\phi \,. \tag{7}$$

Figure 4 shows several TL curves for varying angles of incidence for a 25 mm thick steel plate in air with a 0.005 loss factor, along with the diffuse field incidence computed using Equations 5 and 7. Compared to single angle of incidence TL curves, the diffuse field curve has nearly the same slope at low frequencies (following the mass law), but a much smaller slope above coincidence. The random incidence TL curve above coincidence is an integrated average of all of the dips, reducing the effective slope.

Adding structural damping to a panel has different effects on the TL for a single angle of incidence, and a TL statistically averaged over all incidence angles. Figure 5 shows the same panel from Figure 4, but with damping increased by a factor of 10 to 0.050. The individual coincidence dips are mitigated by the damping, but the TL below (mass controlled) and above (stiffness controlled) coincidence is unaffected. However, the integrated TL (Equation 7) above coincidence increases with added damping, since the integration is over a distribution of individual coincidence dips over all angles. The net effect is more easily seen in Figure 6, which compares the statistically integrated TL curves for the different damping coefficients.

Adding mass to a structural panel does two things: (1) increases the lowest coincidence frequency by slowing down the structural waves, and (2) increases the panel impedance below coincidence. The example in Figure 7 shows the impact of doubling the panel surface mass density.

There are some well-known low and high frequency approximations for the random incidence TL. For low frequencies:

$$TL_d(f < f_c) \cong TL_0 - 10\log_{10}(0.23 TL_0) \tag{8}$$

where TL_0 is $10\log_{10}(1/\tau_{\phi=0})$, or the mass-law normal incidence transmission loss. Since transmission loss is maximized (transmission coefficient is minimized) at normal incidence, the random angle of incidence transmission loss is a fraction of that at normal incidence.

For high frequencies, Cremer's approximation works well:

$$TL_d(f > f_c) \cong TL_0 + 10\log\left(f / f_c - 1\right) + 10\log(\eta) - 2\left(dB\right) \tag{9}$$

where f_c is the lowest coincidence, or critical frequency. Figure 8 compares the low and high frequency approximations with the integrated TL.

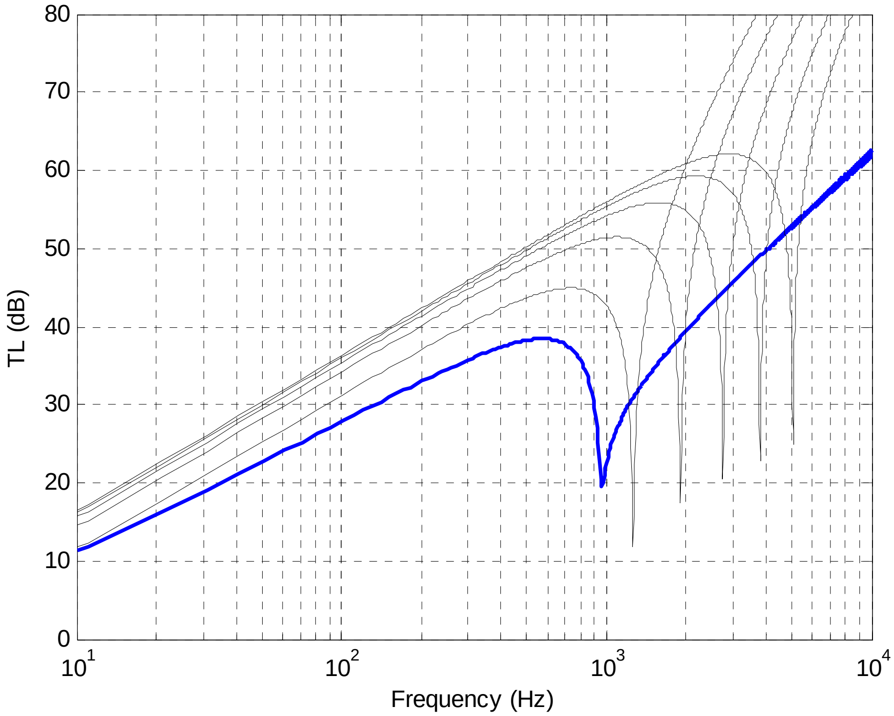

Fig. 4. Random diffuse field incidence TL for a 25 mm thick steel panel in air with loss factor of 0.005 in dark blue, with several TL curves at various angles of incidence in light black.

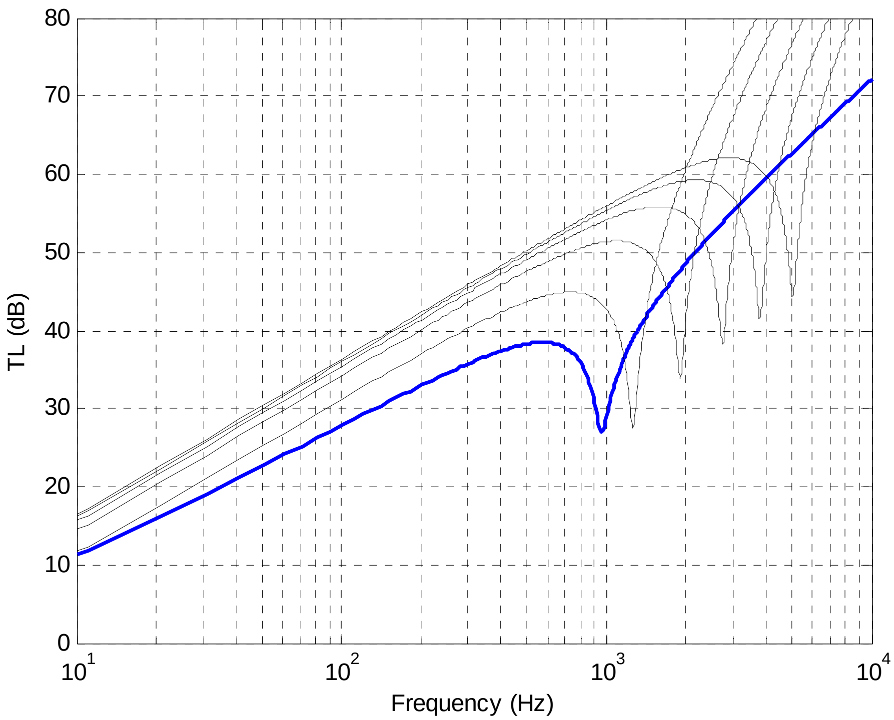

Fig. 5. Random diffuse field incidence TL for a 25 mm thick steel panel in air with loss factor of 0.050 in dark blue, with several TL curves at various angles of incidence in light black.

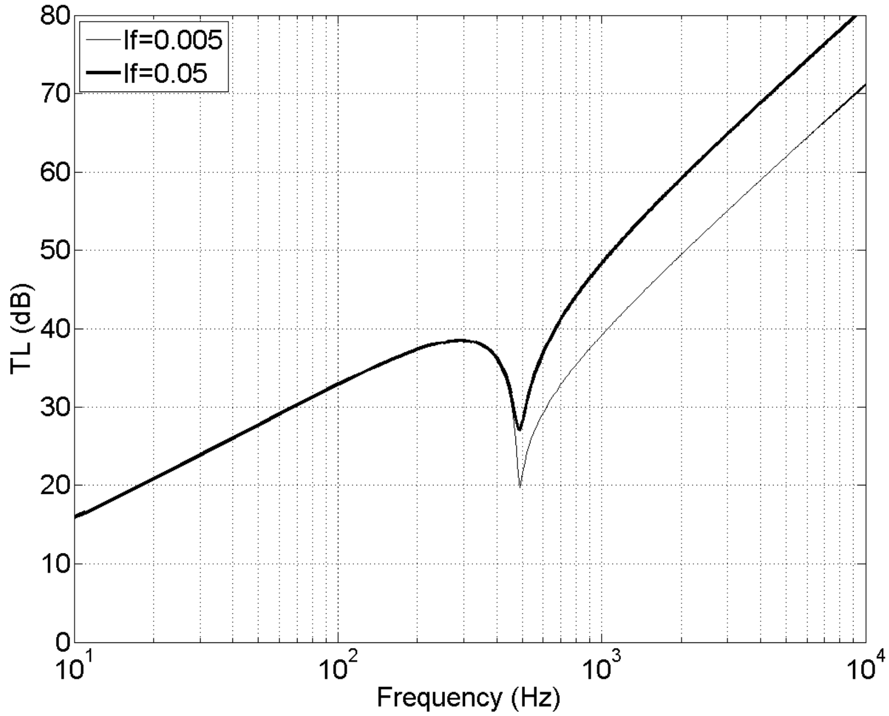

Fig. 6. Random diffuse field incidence TL for a 25 mm thick steel panel in air with variable damping.

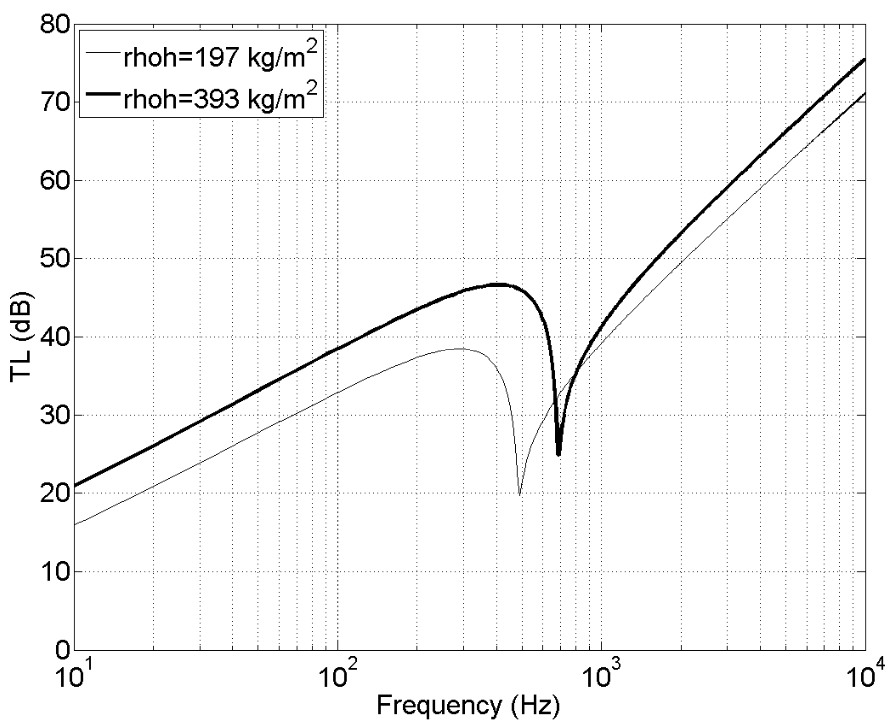

Fig. 7. Random diffuse field incidence TL for a 25 mm thick steel panel in air with variable mass

density.

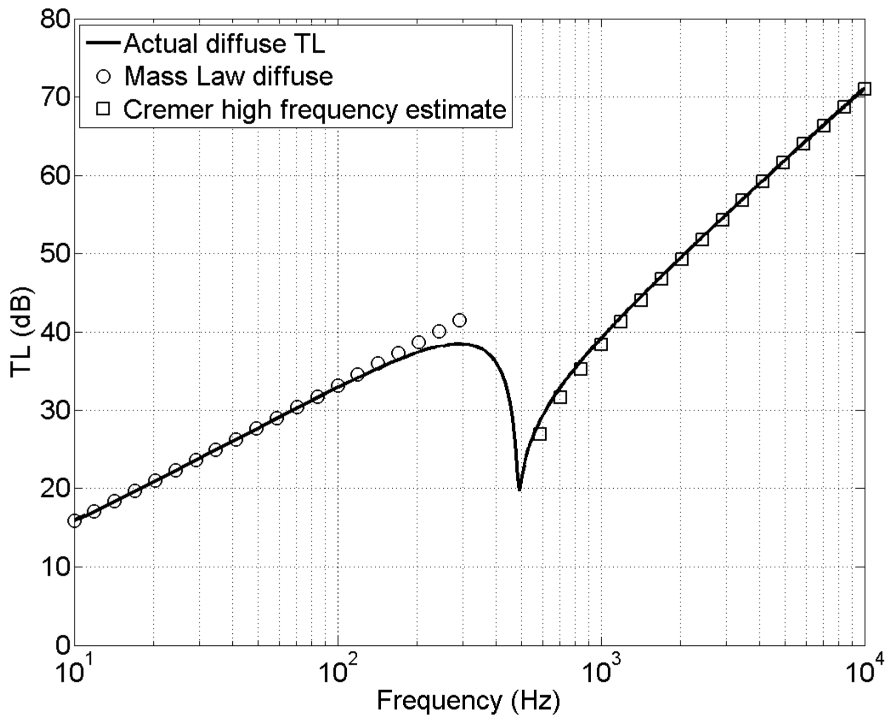

Fig. 8. Random diffuse field incidence TL for a 25 mm thick steel panel with loss factor of 0.005 compared to low and high frequency approximations.

3 FINAL THOUGHTS

Minimizing sound transmission through structures is a common problem in many industries. Here are just a few examples:

- sound from an adjacent room in a building transmitting through walls, ceilings, and floors into other rooms;
- engine noise transmitted through an automobile frame/paneling into the interior;
- rotorcraft transmission noise transmitted through the roof into the interior; and
- rocket pressure pulsations transmitted through the walls of a launch vehicle into a payload interior.

There are many others. These more complex problems often must be solved using numerical methods like finite element and boundary element analysis. However, the basic principles described here should always be kept in mind when diagnosing and mitigating transmission loss problems, and sometimes simple infinite panel theory may be sufficient to analyze complex problems[3]. Remember that panel mass dictates how well low frequency sound transmits through panels, and panel damping most strongly affects high frequency (at and above coincidence) noise.

4 REFERENCES

1. Hambric, S.A., and Fahnline, J.B., “Structural-Acoustics Tutorial – Part 2: Sound-Structure Interaction, *Acoustics Today*, 1 February 2007.
2. Fahy, F., and Gardonio, P., *Sound and Structural Vibration, 2^{nd} Edition*, Academic Press, 2007.
3. Hambric, S.A., Shepherd, M.R., and Schiller, N.H., “Measurements of the air-borne and structure-borne sound power transmission through a quiet honeycomb core sandwich panel for a rotorcraft roof,” *Proceedings of Internoise 2015*, San Francisco, CA, 9-12 August 2015.